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Hamiltonian and path integral formulations of the Dirac–Born–Infeld–Nambu–Goto D1 brane action with and without a dilation field under gauge-fixing

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Abstract. The Hamiltonian and path integral formulations of the Dirac–Born–Infeld–Nambu–Goto D1-brane action with and without a scalar dilation field are investigated under appropriate gauge-fixing.

1 Introduction

The Dirac–Born–Infeld–Nambu–Goto (DBING) action is one of the most important actions in string theories [1–3]. In the present work we study the Hamiltonian and path integral formulations [3,4] of this action describing the D1-brane with and without a scalar dilation field φ under appropriate gauge-fixing conditions (GFCs).

In the next section, the action is considered without the dilation field and in Sect. 3, the action is studied in the presence of a scalar dilation field φ. The Hamiltonian and path integral quantizations are studied in both the cases under appropriate canonical gauge-fixing in the absence of boundary conditions (BCs). Finally a summary and discussion are presented in Sect. 4.

2 The action without a dilation field

We consider the (bosonic) DBING action describing the propagation of a D1-brane (D-string) in a d-dimensional flat background (with $d = 10$ for the fermionic and $d = 26$ for the bosonic D1-brane) defined by [1,2]

$$S_1 = \int \mathcal{L}_1 \, \mathrm{d}^2\sigma, \tag{1a}$$

$$\begin{aligned}
\mathcal{L}_1 &= (-T)[-\det(G_{\alpha\beta} + \mathcal{F}_{\alpha\beta})]^{\frac{1}{2}} && \text{(1b)}\\
&= (-T)[-\det(G_{\alpha\beta} + F_{\alpha\beta} - B_{\alpha\beta})]^{\frac{1}{2}} && \text{(1c)}\\
&= (-T)[-\det(\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + F_{\alpha\beta} - B_{\alpha\beta})]^{\frac{1}{2}} && \text{(1d)}\\
&= (-T) && \text{(1e)}\\
&\quad \times [-\det(\partial_\alpha X^\mu \partial_\beta X_\mu + (\partial_\alpha A_\beta - \partial_\beta A_\alpha) - B_{\alpha\beta})]^{\frac{1}{2}}\\
&= (-T)[(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2 - (\dot{A}_1 - A_0')^2\\
&= 2b(\dot{A}_1 - A_0') - b^2]^{\frac{1}{2}} && \text{(1f)}\\
&\quad +[-T][(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2 - (f-b)^2]^{\frac{1}{2}} && \text{(1g)}\\
&= [-TL] && \text{(1h)}
\end{aligned}$$

$$\begin{aligned}
G_{\alpha\beta} &= \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}; \quad \alpha, \beta = 0, 1, && \text{(1i)}\\
\eta_{\mu\nu} &= \mathrm{diag}(-1, +1, ... + 1);\\
\mu, \nu &= 0, 1, 2, ..., (d-1), && \text{(1j)}\\
L^2 &= [(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2 - (f-b)^2], && \text{(1k)}\\
\mathcal{F}_{\alpha\beta} &= (F_{\alpha\beta} - B_{\alpha\beta}); \quad F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha), && \text{(1l)}\\
B_{\alpha\beta} &:= \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}; \quad B_{\alpha\beta} = \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}, && \text{(1m)}\\
f &= F_{01} = -F_{10} = (\dot{A}_1 - A_0');\\
b &= B_{01} = -B_{10}, && \text{(1n)}\\
\dot{X}^\mu &\equiv \frac{\partial X^\mu}{\partial \tau}; \quad X'^\mu = \frac{\partial X^\mu}{\partial \sigma};\\
\dot{A}_1 &\equiv \frac{\partial A_1}{\partial \tau}; \quad A_0' \equiv \frac{\partial A_0}{\partial \sigma}. && \text{(1o)}
\end{aligned}$$

In the present work we consider only the bosonic D1-brane with $d = 26$ (however, for the corresponding fermionic case one has $d = 10$). Here $\sigma^\alpha \equiv (\tau, \sigma)$ are the two parameters describing the world-sheet (WS). The overdots and primes denote, in general, the derivatives with respect to the WS coordinates τ and σ. The string tension T is a constant of mass dimension two. $G_{\alpha\beta}$ is the induced metric on the WS, and $X^\mu(\tau,\sigma)$ are the maps of the WS into the d-dimensional Minkowski space and describe the string's evolution in space-time [1,2]. Here $F_{\alpha\beta}$ is the Maxwell field strength of the $U(1)$ gauge field $A_\alpha(\tau,\sigma)$, and $B_{\alpha\beta}(\tau,\sigma)$ is a constant background anti-symmetric NSNS 2-form gauge field. It is important to mention here that the 2-form gauge field $B_{\alpha\beta}$ is a scalar

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field in the target space, whereas it is an antisymmetric tensor field in the WS-space. Also, we are considering the 2-form gauge field $B_{\alpha\beta}$ as well as the $U(1)$ gauge fields A_α to be functions only of the WS coordinates τ and σ and not of the target-space coordinates X^μ [1,2]. Further the theory described by the action S_1 is a gauge-invariant (GI) (and consequently a gauge non-anomalous) theory possessing the usual three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) [1,2].

The canonical momenta obtained from $\mathcal{L}_1$ are

$$\begin{aligned} \Pi^\mu &:= \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau X_\mu)} \\ &= [-T/L][(\dot{X}\cdot X')X'^\mu - (X')^2\dot{X}^\mu], \quad (2a) \end{aligned}$$

$$\pi^0 := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau A_0)} = 0, \quad (2b)$$

$$E(\equiv \pi^1) := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau A_1)} = [T/L][f-b], \quad (2c)$$

$$\Pi_b := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau b)} = 0, \quad (2d)$$

$$\partial_\tau \equiv \partial/\partial_\tau; \quad \partial_\sigma \equiv \partial/\partial_\sigma, \quad (2e)$$

where $\Pi^\mu, \pi^0, E(\equiv \pi^1)$ and Π_b are the canonical momenta conjugate respectively to X_μ, A_0, A_1 and $b(= B_{01} = -B_{10})$. The theory described by S_1 is thus seen to possess four primary constraints:

$$\begin{aligned} \psi_1 &= \Pi_b \approx 0, && (3a)\\ \psi_2 &= \pi^0 \approx 0, && (3b)\\ \psi_3 &= (\Pi\cdot X') \approx 0, && (3c)\\ \psi_4 &= [\Pi^2 + (E^2+T^2)(X')^2] \approx 0. && (3d) \end{aligned}$$

Here the symbol $\approx$ denotes a weak equality (WE) in the sense of Dirac [4], and it implies that the above constraints hold as strong equalities only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality (WOE) for the corresponding quantum theory) [4]. The canonical Hamiltonian density corresponding tot $\mathcal{L}_1$ is

$$\begin{aligned} \mathcal{H}_1^{\rm c} &= \Big[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) \\ &\quad + E(\partial_\tau A_1) + \Pi_b(\partial_\tau b) - \mathcal{L}_1\Big] && (4a)\\ &= [EA_0' + Eb]. && (4b) \end{aligned}$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density $\mathcal{H}_1^{\rm c}$ with the help of Lagrange multiplier fields $u_1(\tau,\sigma)$, $u_2(\tau,\sigma)$, $u_3(\tau,\sigma)$ and $u_4(\tau,\sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written

$$\begin{aligned} \mathcal{H}_1^{\rm T} &= [\mathcal{H}_1^{\rm c} + u_1\psi_1 + u_2\psi_2 + u_3\psi_3 + u_4\psi_4] && (5a)\\ &= \Big[EA_0' + Eb + u_1\Pi_b + u_2\pi^0 + u_3(\Pi\cdot X') \\ &\quad + u_4[\Pi^2 + (E^2+T^2)(X')^2]\Big]. && (5b) \end{aligned}$$

We denote the momenta conjugate to u_1, u_2, u_3 and u_4 by $p_{u_1}, p_{u_2}, p_{u_3}$ and p_{u_4} respectively. The Hamilton equations of motion obtained from the total Hamiltonian

$$H_1^{\rm T} = \int \mathcal{H}_1^{\rm T} {\rm d}\sigma, \quad (6)$$

e.g., for the closed strings with the periodic BCs, are

$$\begin{aligned} +\partial_\tau X^\mu &= \frac{\partial H_1^{\rm T}}{\partial \Pi_\mu} = [u_3 X'^\mu + 2\Pi^\mu u_4], && (7a)\\ -\partial_\tau \Pi^\mu &= \frac{\partial H_1^{\rm T}}{\partial X_\mu} \\ &= -\partial_\sigma[u_3\Pi^\mu + 2X'^\mu(E^2+T^2)u_4], && (7b)\\ +\partial_\tau A_0 &= \frac{\partial H_1^{\rm T}}{\partial \pi^0} = u_2, && (7c)\\ -\partial_\tau \pi^0 &= \frac{\partial H_1^{\rm T}}{\partial A_0} = -E', && (7d)\\ +\partial_\tau A_1 &= \frac{\partial H_1^{\rm T}}{\partial E} = [A_0' + b + 2E(X')^2 u_4], && (7e)\\ -\partial_\tau E &= \frac{\partial H_1^{\rm T}}{\partial A_1} = 0, && (7f)\\ +\partial_\tau b &= \frac{\partial H_1^{\rm T}}{\partial \Pi_b} = u_1, && (7g)\\ -\partial_\tau \Pi_b &= \frac{\partial H_1^{\rm T}}{\partial b} = E, && (7h)\\ +\partial_\tau u_1 &= \frac{\partial H_1^{\rm T}}{\partial p_{u_1}} = 0, && (7i)\\ -\partial_\tau p_{u_1} &= \frac{\partial H_1^{\rm T}}{\partial u_1} = \Pi_b, && (7j)\\ +\partial_\tau u_2 &= \frac{\partial H_1^{\rm T}}{\partial p_{u_2}} = 0, && (7k)\\ -\partial_\tau p_{u_2} &= \frac{\partial H_1^{\rm T}}{\partial u_2} = \pi^0, && (7l)\\ +\partial_\tau u_3 &= \frac{\partial H_1^{\rm T}}{\partial p_{u_3}} = 0, && (7m)\\ -\partial_\tau p_{u_3} &= \frac{\partial H_1^{\rm T}}{\partial u_3} = (\Pi\cdot X'), && (7n)\\ +\partial_\tau u_4 &= \frac{\partial H_1^{\rm T}}{\partial p_{u_4}} = 0, && (7o)\\ -\partial_\tau p_{u_4} &= \frac{\partial H_1^{\rm T}}{\partial u} = [\Pi^2 + (E^2+T^2)(X')^2]. && (7p) \end{aligned}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraint ψ_1 be preserved in the course of time one obtains a secondary constraint (with the Poisson bracket (PB) being denoted by $\{\ ,\ \}_{\rm P}$):

$$\tilde{\psi}_5 = \{\psi_1, \mathcal{H}_1^{\rm T}\}_{\rm P} = [-E] \approx 0. \quad (8)$$

The preservation of ψ_2 for all times gives rise to another secondary constraint:

$$\tilde{\psi}_6 = \{\psi_2, \mathcal{H}^{\rm T}\}_{\rm P} = (E') \approx 0. \quad (9)$$

The preservation of $\tilde{\psi}_5$ and $\tilde{\psi}_6$ for all times does not give rise to any further constraints and similarly the preservation of ψ_3 and ψ_4 for all times also does not yield any further constraints.

Further, the constraint $\tilde{\psi}_5$ also implies the constraint $\tilde{\psi}_6$ and therefore it is enough to consider only one constraint, namely

$$\psi_5 = E \approx 0, \tag{10}$$

in the rest of our work (instead of the two constraints $\tilde{\psi}_5$ and $\tilde{\psi}_6$). In view of this, from now onwards, we will consider only one secondary constraint, namely ψ_5 (and not $\tilde{\psi}_5$ and $\tilde{\psi}_6$), in our work. The theory is thus seen to possess only five constraints: ψ_1, ψ_2, ψ_3, ψ_4 and ψ_5. Also the first-order Lagrangian density of the theory is

$$\begin{aligned}\mathcal{L}_1^{\mathrm{IO}} &= \Big[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1) + \Pi_b(\partial_\tau b) \\ &\quad + p_{u_1}(\partial_\tau u_1) + p_{u_2}(\partial_\tau u_2) \\ &\quad + p_{u_3}(\partial_\tau u_3) + p_{u_4}(\partial_\tau u_4) - \mathcal{H}_1^{\mathrm{T}}\Big] \qquad (11a) \\ &= [\Pi^2 + (E^2 - T^2)(X')^2]u_4. \qquad (11b)\end{aligned}$$

The matrix of the Poisson brackets of the constraints ψ_i is seen to be a singular matrix implying that the *set* of constraints ψ_i is first-class [3–5] and that the theory described by S_1 is a gauge-invariant (GI) theory. It is rather well known that the theory described by S_1 indeed possesses three local gauge symmetries given by the two-dimensional WS reparameterization invariance (WSRI) and the Weyl invariance (WI) [1,2].

To study the Hamiltonian and path integral formulations of this GI theory under GFCs, we convert the *set* of first-class constraints of the theory ψ_i into a *set* of second-class constraints, by imposing, arbitrarily, some additional constraints on the system, called the GFCs or the gauge constraints. For this pupose, we could choose, for example, the *set* of GFCs [3–5]

$$\begin{aligned}\psi_6 &= \zeta_1 = X^2 \approx 0, && (12a)\\ \psi_7 &= \zeta_2 = \Pi' \approx 0, && (12b)\\ \psi_8 &= \zeta_3 = A_1 \approx 0, && (12c)\\ \psi_9 &= \zeta_4 = A_0 \approx 0, && (12d)\\ \psi_{10} &= \zeta_5 = b \approx 0. && (12e)\end{aligned}$$

Corresponding to this choice of GFCs, the total *set* of constraints for the theory under which the quantization of the theory could be studied becomes

$$\begin{aligned}\psi_1 &= \Pi_b \approx 0, && (13a)\\ \psi_2 &= \pi^0 \approx 0, && (13b)\\ \psi_3 &= (\Pi \cdot X') \approx 0, && (13c)\\ \psi_4 &= [\Pi^2 + (E^2 + T^2)(X')^2] \approx 0, && (13d)\\ \psi_5 &= E \approx 0, && (13e)\\ \psi_6 &= \zeta_1 = X^2 \approx 0, && (13f)\\ \psi_7 &= \zeta_2 = \Pi' \approx 0, && (13g)\\ \psi_8 &= \zeta_3 = A_1 \approx 0, && (13h)\\ \psi_9 &= \zeta_4 = A_0 \approx 0, && (13i)\\ \psi_{10} &= \zeta_5 = b \approx 0. && (13j)\end{aligned}$$

We now calculate the matrix $M_{\alpha\beta}(:= \{\psi_\alpha, \psi_\beta\}_p)$ of the Poisson brackets of the constraints ψ_i. The non-vanishing elements of the matrix $M_{\alpha\beta}$ are obtained thus:

$$\begin{aligned}M_{1,10} &= -M_{10,1} = [-1]\ \delta\ (\sigma - \sigma'), && (14a)\\ M_{29} &= -M_{92} = [-1]\ \delta\ (\sigma - \sigma'), && (14b)\\ M_{36} &= -M_{63} = [-2X']\ \delta\ (\sigma - \sigma'), && (14c)\\ M_{37} &= -M_{73} = [-\Pi]\ \delta''\ (\sigma - \sigma'), && (14d)\\ M_{46} &= -M_{64} = [-4\Pi]\ \delta\ (\sigma - \sigma'), && (14e)\\ M_{47} &= -M_{74} = [-2X'(E^2 + T^2)]\ \delta''\ (\sigma - \sigma'), && (14f)\\ M_{48} &= -M_{84} = [-2E(X')^2]\ \delta\ (\sigma - \sigma'), && (14g)\\ M_{58} &= -M_{85} = [-1]\ \delta\ (\sigma - \sigma'), && (14h)\\ M_{67} &= -M_{76} = [-2X']\ \delta'\ (\sigma - \sigma'). && (14i)\end{aligned}$$

The matrix $M_{\alpha\beta}$ is seen to be non-singular, implying that the corresponding *set* of constraints ψ_i is a *set* of second-class constraints [3–5]. The determinant of the matrix $M_{\alpha\beta}$ is given by

$$[\|\det(M_{\alpha\beta})\|]^{\frac{1}{2}}, = [4M\delta''(\sigma - \sigma')\delta'(\sigma - \sigma')\delta^4(\sigma - \sigma')], \tag{15a}$$

$$M = [\Pi^2 - (E^2 + T^2)(X')^2]. \tag{15b}$$

The non-vanishing elements of the inverse of the matrix $M_{\alpha\beta}$ (i.e., the elements of the matrix $(M^{-1})_{\alpha\beta}$) are

$$\begin{aligned}(M^{-1})_{1,10} &= -(M^{-1})_{10,1} = \delta\ (\sigma - \sigma'), && (16a)\\ (M^{-1})_{29} &= -(M^{-1})_{92} = \delta\ (\sigma - \sigma'), && (16b)\\ (M^{-1})_{33} &= [(E^2 + T^2)(X')^2\Pi/(M^2)]|\sigma - \sigma'| \\ &\quad \times \delta'(\sigma - \sigma')\delta\ (\sigma - \sigma'), && (16c)\\ (M^{-1})_{34} &= +(M^{-1})_{43} \\ &= [-(\Pi^2 + (E^2 + T^2)(X')^2)X'/(4M^2)] \\ &\quad \times |\sigma - \sigma'|\delta'(\sigma - \sigma')\ \delta\ (\sigma - \sigma'), && (16d)\\ (M^{-1})_{35} &= +(M^{-1})_{53} \\ &= [(\Pi^2 + (E^2 + T^2)(X')^2)E(X')^2X'/(2M^2)] \\ &\quad \times |\sigma - \sigma'|\delta'(\sigma - \sigma')\delta\ (\sigma - \sigma'), && (16e)\\ (M^{-1})_{36} &= -(M^{-1})_{63} \\ &= [-(E^2 + T^2)X'/(2M)]\ \delta\ (\sigma - \sigma'), && (16f)\\ (M^{-1})_{37} &= -(M^{-1})_{73} \\ &= [-\Pi/(2M)]|\sigma - \sigma'|, && (16g)\\ (M^{-1})_{44} &= [\Pi(X')^2/(4M^2)]|\sigma - \sigma'| \\ &\quad \times \delta'(\sigma - \sigma')\delta\ (\sigma - \sigma'), && (16h)\\ (M^{-1})_{45} &= +(M^{-1})_{54} \\ &= [-\Pi E(X')^2(X')^2/(2M^2)] \\ &\quad \times |\sigma - \sigma'|\delta'(\sigma - \sigma')\delta\ (\sigma - \sigma'), && (16i)\\ (M^{-1})_{46} &= -(M^{-1})_{64} \\ &= [\Pi/(4M)]\delta\ (\sigma - \sigma'), && (16j)\\ (M^{-1})_{47} &= -(M^{-1})_{74} = [-(X')/(4M)]|\sigma - \sigma'|, && (16k)\\ (M^{-1})_{55} &= [\Pi E^2(X')^2)(X')^2(X')^2/(M^2)]|\sigma - \sigma'|\end{aligned}$$

$$\times\, \delta'(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{16l}$$

$$(M^{-1})_{56} = -(M^{-1})_{65} = [-\Pi E(X')^2/(2M)]\ \delta\,(\sigma-\sigma'), \tag{16m}$$

$$(M^{-1})_{57} = -(M^{-1})_{75} = [E(X')^2(X')/(2M)]|\sigma-\sigma'|, \tag{16n}$$

$$(M^{-1})_{58} = -(M^{-1})_{85} = \delta\,(\sigma-\sigma'), \tag{16o}$$

with the step functions $\epsilon(\sigma-\sigma')$ defined by

$$\epsilon(\sigma-\sigma') := \begin{cases} +1, (\sigma-\sigma')>0, \\ -1, (\sigma-\sigma')<0, \end{cases} \tag{17}$$

and

$$\int M(\sigma,\sigma'')M^{-1}(\sigma'',\sigma')\mathrm{d}\sigma'' = \mathbf{1}_{10\times 10}\delta(\sigma-\sigma'). \tag{18}$$

Now following the standard Dirac quantization procedure in the Hamiltonian formulation [4], the non-vanishing equal WS time (EWST) Dirac brackets (DBs) (denoted by $\{\ ,\ \}_\mathrm{D}$) of the theory described by the action S_1 under the GFCs ζ_i are obtained (with the arguments of the variables being suppressed) as [3–5]

$$\begin{aligned}
&\{X^\mu, X_\nu\}_\mathrm{D} \\
&= [1/(2M^2)]\Big[(E^2+T^2)(X')^2(\Pi X'^\mu X'_\nu) \\
&- X'X'^\mu\Pi_\nu - X'\Pi^\mu X'_\nu \\
&+ 2\Pi(X')^2\Pi^\mu\Pi_\nu - \Pi^2(X')(X'^\mu\Pi_\nu + \Pi^\mu X'_\nu)\Big]|\sigma-\sigma'| \\
&+ [1/(2M)]\Big[2X'\Pi^\mu X_\nu \\
&- \Pi(X'^\mu X_\nu + X^\mu X'_\nu)\Big]\epsilon(\sigma-\sigma'),
\end{aligned} \tag{19a}$$

$$\begin{aligned}
&\{\Pi^\mu, \Pi_\nu\}_\mathrm{D} \\
&= [[1/M^2]\Big[(E^2+T^2)[\Pi^2+(E^2+T^2)(X')^2](X') \\
&\times(\Pi^\mu X'_\nu + X'^\mu\Pi_\nu) \\
&- \Pi(E^2+T^2)(X')^2(\Pi^\mu\Pi_\nu + 2X'^\mu X'_\nu)\Big]\delta(\sigma-\sigma') \\
&+ [1/M]\Big[\Pi(E^2+T^2) \\
&\times(X'^\mu X_\nu + X^\mu X'_\nu - \Pi^\mu X_\nu - X^\mu\Pi_\nu)\Big]\delta'(\sigma-\sigma') \\
&+ [1/(2M)]\Big[\Pi(\Pi^\mu\Pi'_\nu + \Pi'^\mu\Pi_\nu) \\
&- \Pi(E^2+T^2)(X'^\mu\Pi'_\nu + \Pi'^\mu X'_\nu)\Big]\epsilon(\sigma-\sigma'),
\end{aligned} \tag{19b}$$

$$\{X^\mu, \Pi_\nu\}_\mathrm{D} = [-\delta^\mu_\nu]\delta(\sigma-\sigma'), \tag{19c}$$

$$\{A_1, A_1\}_\mathrm{D} = [2/M^2][E^2(X')^2(X')^2(X')^2\Pi]|\sigma-\sigma'|, \tag{19d}$$

$$\begin{aligned}
&\{A_1, X^\mu\}_\mathrm{D} \\
&= [[1/(2M^2)]\Big[2E\Pi(X')^2(X')^2\Pi^\mu \\
&- [\Pi^2+(E^2+T^2)(X')^2](X')X'^\mu\epsilon(\sigma-\sigma')\Big] \\
&- [1/(2M)][EX'^2(X')X^\mu]\epsilon(\sigma-\sigma').
\end{aligned} \tag{19e}$$

It is important to recall here that the constraints of the theory represent only the weak equalities in the sense of Dirac [4], as explained in the foregoing, implying that they are strongly zero only on the reduced hypersurface of the constraints and not in the rest of the phase space of the (classical) theory (with a similar weak operator equality holding for the corresponding quantum theory) and as a consequence of this the DBs involving the gauge fields like A_α (or $B_{\alpha\beta}$) can indeed be non-vanishing in principle (as is evident in the present case from the above results) which would, however, become strongly zero on the reduced hypersurface of the constraints of the theory described by the action in any case.

Further, in the canonical quantization of the theory, while going from equal WS time (EWST) Dirac brackets of the theory to the corresponding EWST commutation relations, one would encounter here the problem of operator ordering [6] because the product of canonical variables of the theory are involved in the classical description of the theory (like in the expressions for the constraints of the theory) as well as in the calculation of the Dirac brackets. These variables are envisaged as non-commuting operators in the quantized theory leading to the problem of so-called operator ordering [6]. This problem could, however, be resolved [6] by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators [6].

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z_1[J_i]$ of the theory under GFCs ζ_i in the presence of the external sources J_i (following the Senjanovic procedure [2,3] for a theory possessing a set of second-class constraints, appropriate for our theory described by the action S_1 considered under the GFCs ζ_i (12) [2,3]) as follows:

$$\begin{aligned}
&Z_1[J_i] \\
&= \int[\mathrm{d}\mu]\exp\Bigg[\mathrm{i}\int\mathrm{d}^2\sigma[J_i\Phi^i + \Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) \\
&+ E(\partial_\tau A_1) + \Pi_b(\partial_\tau b) \\
&+ p_{u_1}(\partial_\tau u_1) + p_{u_2}(\partial_\tau u_2) + p_{u_3}(\partial_\tau u_3) \\
&+ p_{u_4}(\partial_\tau u_4) - \mathcal{H}_1^\mathrm{T}]\Bigg],
\end{aligned} \tag{20}$$

where the phase-space variables of the theory are $\Phi^i \equiv (X^\mu, A_0, A_1, b, u_1, u_2, u_3, u_4)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_\mu, \pi^0, E, \Pi_b, p_{u_1}, p_{u_2}, p_{u_3}, p_{u_4})$. The functional measure $[\mathrm{d}\mu]$ of the generating functional $Z_1[J_i]$ under the GFCs ζ_i is obtained using (11), (13), (15) and (20):

$$\begin{aligned}
[\mathrm{d}\mu] &= [4M\delta''(\sigma-\sigma')\delta'(\sigma-\sigma')\delta^3(\sigma-\sigma')] \\
&\times [\mathrm{d}X^\mu][\mathrm{d}A_0][\mathrm{d}A_1][[\mathrm{d}b][\mathrm{d}u_1][\mathrm{d}u_2][\mathrm{d}u_3][\mathrm{d}u_4] \\
&\times [\mathrm{d}\Pi_\mu][\mathrm{d}\pi^0][\mathrm{d}E][\mathrm{d}\Pi_b][\mathrm{d}p_{u_1}][\mathrm{d}p_{u_2}][\mathrm{d}p_{u_3}][\mathrm{d}p_{u_4}]
\end{aligned}$$

$$\times\ \delta[(\Pi_b)\approx 0]\cdot\delta[(\pi^0)\approx 0]\cdot\delta(\Pi\cdot X')\approx 0]\cdot$$
$$\times\ \delta[[\Pi^2+(E^2+T^2)(X')^2]\approx 0]$$
$$\times\ \delta[(E)\approx 0]\cdot\delta[(X^2)\approx 0]$$
$$\times\ \delta[(\Pi')\approx 0]\delta[(A_1)\approx 0]$$
$$\times\ \delta[(A_0)\approx 0]\cdot\delta[(b)\approx 0]. \quad (21)$$

The Hamiltonian and path integral quantization of the theory described by the action S_1 under the GFCs ζ_i is now complete. In the next section we study this theory in the presence of a scalar dilation field.

3 The action in the presence of a scalar dilation field

The (bosonic) DBING action describing the propagation of a D1-brane (D-string) in a d-dimensional flat background in the presence of a scalar dilation field φ is defined by [1,2]

$$S_2 = \int \mathcal{L}_2 \mathrm{d}^2\sigma, \quad (22a)$$
$$\mathcal{L}_2 = [\mathrm{e}^{-\varphi}\mathcal{L}_1] \quad (22b)$$
$$= [-TL\mathrm{e}^{-\varphi}]. \quad (22c)$$

The canonical momenta obtained from $\mathcal{L}_2$ are

$$\Pi^\mu := \frac{\partial \mathcal{L}_2}{\partial(\partial_\tau X_\mu)} \quad (23a)$$
$$= [-T\mathrm{e}^{-\varphi}/L][(\dot{X}\cdot X')X'^\mu - (X')^2\dot{X}^\mu],$$
$$\pi^0 := \frac{\partial \mathcal{L}_2}{\partial(\partial_\tau A_0)} = 0, \quad (23b)$$
$$E(\equiv \pi^1) := \frac{\partial \mathcal{L}_2}{\partial(\partial_\tau A_1)} = [T\mathrm{e}^{-\varphi}/L](f-b), \quad (23c)$$
$$\Pi_b := \frac{\partial \mathcal{L}_2}{\partial(\partial_\tau b)} = 0, \quad (23d)$$
$$\pi := \frac{\partial \mathcal{L}_2}{\partial(\partial_\tau \varphi)} = 0. \quad (23e)$$

Here π is the momentum canonically conjugate to the dilation field φ. The theory described by S_2 is thus seen to possess five primary constraints:

$$\chi_1 = \pi \approx 0, \quad (24a)$$
$$\chi_2 = \Pi_b \approx 0, \quad (24b)$$
$$\chi_3 = \pi^0 \approx 0, \quad (24c)$$
$$\chi_4 = (\Pi\cdot X') \approx 0, \quad (24d)$$
$$\chi_5 = [\Pi^2 + (E^2 + T^2\mathrm{e}^{-2\varphi})(X')^2] \approx 0. \quad (24e)$$

The canonical Hamiltonian density corresponding to $\mathcal{L}_2$ is

$$\mathcal{H}_2^{\mathrm{c}} = \Big[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1)$$
$$+ \Pi_b(\partial_\tau b) + \pi(\partial_\tau\varphi) - \mathcal{L}_2\Big] \quad (25a)$$
$$= [E(A_0' + b)]. \quad (25b)$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density of the theory $\mathcal{H}_2^{\mathrm{c}}$ with the help of Lagrange multiplier fields $v_1(\tau,\sigma)$, $v_2(\tau,\sigma), v_3(\tau,\sigma), v_4(\tau,\sigma)$ and $v_5(\tau,\sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written

$$\mathcal{H}_2^{\mathrm{T}}$$
$$= [\mathcal{H}_2^{\mathrm{c}} + v_1\chi_1 + v_2\chi_2 + v_3\chi_3 + v_4\chi_4 + v_5\chi_5] \quad (26a)$$
$$= [E(A_0'+b) + v_1\pi + v_2\Pi_b + v_3\pi^0 + v_4(\Pi\cdot X')$$
$$+ v_5[\Pi^2 + (E^2 + T^2\mathrm{e}^{-2\varphi})(X')^2]]. \quad (26b)$$

We denote the momenta conjugate to v_1, v_2, v_3, v_4 and v_5 by $p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}$ and p_{v_5} respectively. The Hamilton equation of motion obtained from the total Hamiltonian:

$$H_2^{\mathrm{T}} = \int \mathcal{H}_2^{\mathrm{T}}\mathrm{d}\sigma, \quad (27)$$

e.g. for the closed strings with periodic BCs, are

$$+\partial_\tau X^\mu = \frac{\partial H_2^{\mathrm{T}}}{\partial \Pi_\mu} = [v_4 X'^\mu + 2\Pi^\mu v_5], \quad (28a)$$
$$-\partial_\tau \Pi^\mu = \frac{\partial H_2^{\mathrm{T}}}{\partial X_\mu} = -\partial_\sigma[v_4\Pi^\mu + 2X'^\mu(E^2 + T^2\mathrm{e}^{-2\varphi})v_5], \quad (28b)$$
$$+\partial_\tau A_0 = \frac{\partial H_2^{\mathrm{T}}}{\partial \pi^0} = v_3, \quad (28c)$$
$$-\partial_\tau \pi^0 = \frac{\partial H_2^{\mathrm{T}}}{\partial A_0} = [-E'], \quad (28d)$$
$$+\partial_\tau A_1 = \frac{\partial H_2^{\mathrm{T}}}{\partial E} = [A_0' + b + 2E(X')^2 v_5], \quad (28e)$$
$$-\partial_\tau E = \frac{\partial H_2^{\mathrm{T}}}{\partial A_1} = 0, \quad (28f)$$
$$+\partial_\tau b = \frac{\partial H_2^{\mathrm{T}}}{\partial \Pi_b} = v_2, \quad (28g)$$
$$-\partial_\tau \Pi_b = \frac{\partial H_2^{\mathrm{T}}}{\partial b} = E, \quad (28h)$$
$$+\partial_\tau \varphi = \frac{\partial H_2^{\mathrm{T}}}{\partial \pi} = v_1, \quad (28i)$$
$$-\partial_\tau \pi = \frac{\partial H_2^{\mathrm{T}}}{\partial \varphi} = [-2T^2\mathrm{e}^{-2\varphi}(X')^2 v_5], \quad (28j)$$
$$+\partial_\tau v_1 = \frac{\partial H_2^{\mathrm{T}}}{\partial p_{v_1}} = 0, \quad (28k)$$
$$-\partial_\tau p_{v_1} = \frac{\partial H_2^{\mathrm{T}}}{\partial v_1} = \pi, \quad (28l)$$
$$+\partial_\tau v_2 = \frac{\partial H_2^{\mathrm{T}}}{\partial p_{v_2}} = 0, \quad (28m)$$
$$-\partial \tau p_{v_2} = \frac{\partial H_2^{\mathrm{T}}}{\partial v_2} = \Pi_b, \quad (28n)$$
$$+\partial_\tau v_3 = \frac{\partial H_2^{\mathrm{T}}}{\partial p_{v_3}} = 0, \quad (28o)$$
$$-\partial_\tau p_{v_3} = \frac{\partial H_2^{\mathrm{T}}}{\partial v_3} = \pi^0, \quad (28p)$$

$$+\partial_\tau v_4 = \frac{\partial H_2^{\mathrm{T}}}{\partial p_{v_4}} = 0, \tag{28q}$$

$$-\partial_\tau p_{v_4} = \frac{\partial H_2^{\mathrm{T}}}{\partial v_4} = [\Pi \cdot X'], \tag{28r}$$

$$+\partial_\tau v_5 = \frac{\partial H_2^{\mathrm{T}}}{\partial p_{v_5}} = 0, \tag{28s}$$

$$-\partial_\tau p_{v_5} = \frac{\partial H_2^{\mathrm{T}}}{\partial v_5} = [\Pi^2 + (E^2 + T^2 \mathrm{e}^{-2\varphi})(X')^2]. \tag{28t}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraint χ_2 be preserved in the course of time one obtains a secondary constraint (with the Poisson bracket (PB) being denoted by $\{\ ,\ \}_{\mathrm{P}}$):

$$\tilde{\chi}_6 = \{\chi_2, \mathcal{H}_2^{\mathrm{T}}\}_{\mathrm{P}} = [-E] \approx 0. \tag{29}$$

The presentation of χ_3 for all times gives rise to another secondary constraint:

$$\tilde{\chi}_7 = \{\chi_3, \mathcal{H}_2^{\mathrm{T}}\}_p = (E') \approx 0. \tag{30}$$

The preservation of $\tilde{\chi}_6$ and $\tilde{\chi}_7$ for all times does not give rise to any further constraints and similarly the preservation of χ_1, χ_4 and χ_5 for all times does not yield any further constraints.

Further the constraint $\tilde{\chi}_6$ also implies the constraint $\tilde{\chi}_7$ and therefore it is enough to consider only one constraint, namely,

$$\chi_6 = E \approx 0 \tag{31}$$

in the rest of our work (instead of the two constraints $\tilde{\chi}_6$ and $\tilde{\chi}_7$). In view of this, from now onwards we will consider only one secondary constraint, namely χ_6 (and not $\tilde{\chi}_6$ and $\tilde{\chi}_7$) in our work. The theory is thus seen to possess only six constraints: $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ and χ_6. Also the first-order Lagrangian density of the theory is

$$\begin{aligned} \mathcal{L}_2^{\mathrm{IO}} = \Big[& \Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) \\ & + E(\partial_\tau A_1) + \Pi_b(\partial_\tau b) + \pi(\partial_\tau \varphi) \\ & + p_{v_1}(\partial_\tau v_1) + p_{v_2}(\partial_\tau v_2) + p_{v_3}(\partial_\tau v_3) + p_{v_4}(\partial_\tau v_4) \\ & + p_{v_5}(\partial_\tau v_5) - \mathcal{H}_2^{\mathrm{T}} \Big] \qquad (32a) \\ = & [\Pi^2 + (E^2 - T^2 \mathrm{e}^{-2\varphi})(X')^2] v_5. \qquad (32b) \end{aligned}$$

The matrix of the Poisson brackets of the constraints χ_i is seen to be a singular matrix, implying that the set of constraints χ_i is first-class [3–5] and that the theory described by S_2 is a gauge-invariant (GI) theory. It is rather well known that the theory described by S_2 indeed possesses three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) [1,2].

To study the Hamiltonian and path integral formulations of this GI theory under GFCs, we convert the *set* of first-class constraints of the theory χ_i into a *set* of second-class constraints, by imposing, arbitrarily, some additional constraints on the system called the GFCs or the gauge constraints. For this purpose, we could choose, for example, the *set* of GFCs [3–5]

$$\chi_7 = \rho_1 = X^2 \approx 0, \tag{33a}$$
$$\chi_8 = \rho_2 = \Pi' \approx 0, \tag{33b}$$
$$\chi_9 = \rho_3 = A_1 \approx 0, \tag{33c}$$
$$\chi_{10} = \rho_4 = A_0 \approx 0, \tag{33d}$$
$$\chi_{11} = \rho_5 = b \approx 0, \tag{33e}$$
$$\chi_{12} = \rho_6 = \varphi \approx 0. \tag{33f}$$

Corresponding to this choice of GFCs, the total *set* of constraints of the theory under which the quantization of the theory could be studied becomes

$$\chi_1 = \pi \approx 0, \tag{34a}$$
$$\chi_2 = \Pi_b \approx 0, \tag{34b}$$
$$\chi_3 = \pi^0 \approx 0, \tag{34c}$$
$$\chi_4 = (\Pi \cdot X') \approx 0, \tag{34d}$$
$$\chi_5 = [\Pi^2 + (E^2 + T^2 \mathrm{e}^{-2\varphi}(X')^2] \approx 0, \tag{34e}$$
$$\chi_6 = E \approx 0, \tag{34f}$$
$$\chi_7 = \rho_1 = X^2 \approx 0, \tag{34g}$$
$$\chi_8 = \rho_2 = \Pi' \approx 0, \tag{34h}$$
$$\chi_9 = \rho_3 = A_1 \approx 0, \tag{34i}$$
$$\chi_{10} = \rho_4 = A_0 \approx 0, \tag{34j}$$
$$\chi_{11} = \rho_5 = b \approx 0, \tag{34k}$$
$$\chi_{12} = \rho_6 = \varphi \approx 0. \tag{34l}$$

We now calculate the matrix $R_{\alpha\beta} (:= \{\chi_\alpha, \chi_\beta\}_{\mathrm{P}})$ of the Poisson brackets of the constraints χ_i. The non-vanishing elements of the matrix $R_{\alpha\beta}$ are obtained thus:

$$R_{15} = -R_{51} = [2T^2 \mathrm{e}^{-2\varphi}(X')^2]\ \delta\ (\sigma - \sigma'), \tag{35a}$$
$$R_{1,12} = -R_{12,1} = [-1]\ \delta(\sigma - \sigma'), \tag{35b}$$
$$R_{2,11} = -R_{11,2} = [-1]\ \delta(\sigma - \sigma'), \tag{35c}$$
$$R_{3,10} = -R_{10,3} = [-1]\ \delta(\sigma - \sigma'), \tag{35d}$$
$$R_{47} = -R_{74} = [-2X']\ \delta(\sigma - \sigma'), \tag{35e}$$
$$R_{48} = -R_{84} = [-\Pi]\ \delta''\ (\sigma - \sigma'), \tag{35f}$$
$$R_{57} = -R_{75} = [-4\Pi]\ \delta(\sigma - \sigma'), \tag{35g}$$
$$R_{58} = -R_{85} = [-2(X')(E^2 + T^2 \mathrm{e}^{-2\varphi}]\ \delta''\ (\sigma - \sigma'), \tag{35h}$$
$$R_{59} = -R_{95} = [-2E(X')^2]\ \delta(\sigma - \sigma'), \tag{35i}$$
$$R_{69} = -R_{96} = [-1]\ \delta(\sigma - \sigma'), \tag{35j}$$
$$R_{78} = +R_{87} = [-2X']\ \delta'\ (\sigma - \sigma'). \tag{35k}$$

The matrix $R_{\alpha\beta}$ is seen to be non-singular, implying that the corresponding *set* of constraints χ_i is a *set* of second-class constraints [5]. The determinant of the matrix $R_{\alpha\beta}$ is given by

$$[\| \det(R_{\alpha\beta}) \|]^{\frac{1}{2}} = [4R\delta''(\sigma - \sigma')\ \delta^5(\sigma - \sigma')], \tag{36a}$$
$$R = [\Pi^2 - (E^2 + T^2 \mathrm{e}^{-2\varphi})(X')^2]. \tag{36b}$$

The non-vanishing elements of the inverse of the matrix $R_{\alpha\beta}$ (i.e., the elements of the matrix $(R^{-1})_{\alpha\beta}$) are

$$(R^{-1})_{1,12} = -(R^{-1})_{12,1} = \delta\ (\sigma - \sigma'), \tag{37a}$$

$$(R^{-1})_{2,11} = -(R^{-1})_{11,2} = \delta\,(\sigma-\sigma'), \tag{37b}$$
$$(R^{-1})_{3,10} = -(R^{-1})_{10,3} = \delta\,(\sigma-\sigma'), \tag{37c}$$
$$(R^{-1})_{44} = [2\Pi(X')^2(E^2+T^2\mathrm{e}^{-2\varphi})/(2R^2)] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{37d}$$
$$(R^{-1})_{45} = +(R^{-1})_{54} = [-(X')(\Pi^2+(E^2+T^2\mathrm{e}^{-2\varphi})(X')^2/(4R^2)] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{37e}$$
$$(R^{-1})_{46} = +(R^{-1})_{64} = \Big[E(X')(X')^2 \times (\Pi^2+(E^2+T^2\mathrm{e}^{-2\varphi})(X')^2/(2R^2)\Big] \times |\sigma-\sigma'|\;\delta(\sigma-\sigma'), \tag{37f}$$
$$(R^{-1})_{47} = -(R^{-1})_{74} = [(X')(E^2+T^2\mathrm{e}^{-2\varphi})/(2R)]\delta\,(\sigma-\sigma'), \tag{37g}$$
$$(R^{-1})_{48} = -(R^{-1})_{84} = [-\Pi/(2R)]|\sigma-\sigma'|, \tag{37h}$$
$$(R^{-1})_{4,12} = +(R^{-1})_{12,4} = \Big[-(X')(X')^2(T^2\mathrm{e}^{-2\varphi}) \times (\Pi^2+(E^2+T^2\mathrm{e}^{-2\varphi})(X')^2)/(2R^2)\Big] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{37i}$$
$$(R^{-1})_{55} = [\Pi(X')^2/(4R^2)]|\sigma-\sigma'| \times \delta'(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{37j}$$
$$(R^{-1})_{56} = +(R^{-1})_{65} = [-\Pi E(X')^2(X')^2/(2R^2)]\;\;|\,\sigma-\sigma'\,| \times \delta^{'}\;(\sigma-\sigma')\delta\,(\sigma-\sigma'), \tag{37k}$$
$$(R^{-1})_{57} = -(R^{-1})_{75} = [-\Pi/(4R)]\;\delta\,(\sigma-\sigma'), \tag{37l}$$
$$(R^{-1})_{58} = -(R^{-1})_{85} = [(X')/(4R)]|\sigma-\sigma'|, \tag{37m}$$
$$(R^{-1})_{5,12} = +(R^{-1})_{12,5} = [\Pi(X')^2(X')^2(T^2\mathrm{e}^{-2\varphi})/(2R^2)] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\;\delta\,(\sigma-\sigma'), \tag{37n}$$
$$(R^{-1})_{66} = [2\Pi E^2(X')^2(X')^2(X')^2/(2R^2)] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\;\delta\,(\sigma-\sigma'), \tag{37o}$$
$$(R^{-1})_{67} = -(R^{-1})_{76} = [\Pi E(X')^2/(2R)]\;\delta\,(\sigma-\sigma'), \tag{37p}$$
$$(R^{-1})_{68} = -(R^{-1})_{86} = [-E(X')(X')^2/(2R)]|\sigma-\sigma'|, \tag{37q}$$
$$(R^{-1})_{69} = -(R^{-1})_{96} = \delta\,(\sigma-\sigma'), \tag{37r}$$
$$(R^{-1})_{6,12} = +(R^{-1})_{12,6} = [-2\Pi E(X')^2(X')^2(X')^2(T^2\mathrm{e}^{-2\varphi})/(2R^2)] \times |\sigma-\sigma'|\;\delta'(\sigma-\sigma')\;\delta\,(\sigma-\sigma'), \tag{37s}$$
$$(R^{-1})_{7,12} = -(R^{-1})_{12,7} = [\Pi(X')^2(T^2\mathrm{e}^{-2\varphi})/(2R)]\;\delta\,(\sigma-\sigma'), \tag{37t}$$
$$(R^{-1})_{8,12} = -(R^{-1})_{12,8} = [-(X')(X')^2(T^2\mathrm{e}^{-2\varphi})/(2R)]|\sigma-\sigma'|, \tag{37u}$$

with

$$\int R(\sigma,\sigma'')R^{-1}(\sigma'',\sigma')\mathrm{d}\sigma'' = \mathbf{1}_{12\times12}\;\delta(\sigma-\sigma'). \tag{38}$$

Now, following the standard Dirac quantization procedure in the Hamiltonian formulation [4], the non-vanishing EWST Dirac brackets (denoted by $\{\;,\;\}_{\mathrm{D}}$) of the theory in the presence of a scalar dilation field described by the action S_2 under the GFCs ρ_i are obtained (with the arguments of the field variables being suppressed) (with $t := T\mathrm{e}^{-\varphi}$) [3–5]:

$$\begin{aligned}\{X^\mu, X_\nu\}_{\mathrm{D}} &= [1/(2M^2)] \times \Big[(E^2+t^2)(X')^2(\Pi X'^\mu X'_\nu) - X'X'^\mu\Pi_\nu - X'\Pi^\mu X'_\nu \\ &\quad + 2\Pi(X')^2\Pi^\mu\Pi_\nu - \Pi^2(X')(X'^\mu\Pi_\nu + \Pi^\mu X'_\nu)\Big]|\sigma-\sigma'| \\ &\quad + [1/(2M)] \times [2X'\Pi^\mu X_\nu - \Pi(X'^\mu X_\nu + X^\mu X'_\nu)]\epsilon(\sigma-\sigma'),\end{aligned} \tag{39a}$$

$$\begin{aligned}\{\Pi^\mu, \Pi_\nu\}_{\mathrm{D}} &= [1/M^2]\Big[(E^2+t^2) \times [\Pi^2+(E^2+t^2)(X')^2](X')(\Pi^\mu X'_\nu + X'^\mu\Pi_\nu) \\ &\quad - \Pi(E^2+t^2)(X')^2(\Pi^\mu\Pi_\nu + 2X'^\mu X'_\nu)\Big]\delta(\sigma-\sigma') \\ &\quad + [1/M]\Big[\Pi(E^2+t^2)\Big(X'^\mu X_\nu + X^\mu X'_\nu - \Pi^\mu X_\nu - X^\mu\Pi_\nu\Big)\Big]\delta'(\sigma-\sigma') \\ &\quad + [1/(2M)]\Big[\Pi(\Pi^\mu\Pi'_\nu + \Pi'^\mu\Pi_\nu) - \Pi(E^2+t^2)(X'^\mu\Pi'_\nu + \Pi'^\mu X'_\nu)\Big]\epsilon(\sigma-\sigma'),\end{aligned} \tag{39b}$$

$$\{X^\mu, \Pi_\nu\}_{\mathrm{D}} = [-\delta^\mu_\nu]\delta(\sigma-\sigma'), \tag{39c}$$

$$\{A_1, A_1\}_{\mathrm{D}} = [2/M^2][E^2(X')^2(X')^2(X')^2\Pi]|\sigma-\sigma'|, \tag{39d}$$

$$\begin{aligned}\{A_1, X^\mu\}_{\mathrm{D}} &= [1/(2M^2)]\Big[2E\Pi(X')^2(X')^2\Pi^\mu \\ &\quad - [\Pi^2+(E^2+t^2)(X')^2](X')X'^\mu\epsilon(\sigma-\sigma')\Big] \\ &\quad - [1/(2M)][EX'^2(X')X^\mu]\epsilon(\sigma-\sigma'),\end{aligned} \tag{39e}$$

$$\begin{aligned}\{A_1, \Pi^\mu\}_{\mathrm{D}} &= \Big[[1/(M^2)](E^2+t^2) \times [\Pi^2+(E^2+t^2)(X')^2](X')(X')\Pi^\mu \\ &\quad - 2\Pi(E^2+t^2)^2(X')^2(X')X'^\mu\delta(\sigma-\sigma') \\ &\quad + [1/M][(E^2+t^2)\Pi(X')X^\mu]\delta'(\sigma-\sigma')\Big] \\ &\quad - [1/(2M)][(E^2+t^2)(X')(X')\Pi'^\mu]\epsilon(\sigma-\sigma'),\end{aligned} \tag{39f}$$

$$\{X^\mu, \pi\}_{\mathrm{D}} = \Big[[1/(2M)][2\Pi+(X')^2][(t^2(X')^2X^\mu]\epsilon(\sigma-\sigma')$$

$$- [1/(2M^2)][\Pi^2 t^2 (X')^2 (X') X'^\mu] |\sigma - \sigma'| \Big], \quad (39g)$$

$$\{\pi, \Pi^\mu\}_\mathrm{D} = \Big[[1/(2M^2)][\Pi^\mu X' \Pi^2 t^2 (X')^2] \epsilon(\sigma - \sigma')$$
$$+ [1/(2M)][(1 - X') X' (\Pi'^\mu) t^2 (X')^2] |\sigma - \sigma'| \Big], \quad (39h)$$

$$\{\pi, A_1\}_\mathrm{D} = [1/M^2][(X')^2 (X')^2 (X')^2 t^2 \Pi] |\sigma - \sigma'|. \quad (39i)$$

As explained in the previous section, the non-vanishing DBs involving the gauge field A_1, in the above results, would become strongly zero on the reduced hypersurface of the constraints of the theory described by the action S_2 [1,2]. The problem of operator ordering occurring here while making a transition from the EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Sect. 3, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators [6].

In the path integral formulation, the transition to quantum theory is made again by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z_2[J_i]$ of the theory, following again the Senjanovic procedure for a theory possessing a set of second-class constraints [2,3], appropriate for our theory described by the action S_2 considered under the GFCs ρ_i, in the presence of the external sources J_i as follows [2,3]:

$$Z_2[J_i] = \int [\mathrm{d}\mu] \exp \left[\mathrm{i} \int \mathrm{d}^2\sigma \Big[J_i \Phi^i + \Pi^\mu (\partial_\tau X_\mu) \right.$$
$$+ \pi^0 (\partial_\tau A_0) + E (\partial_\tau A_1) + \Pi_b (\partial_\tau b)$$
$$+ \pi (\partial_\tau \varphi) + p_{v_1} (\partial_\tau v_1) + p_{v_2} (\partial_\tau v_2) + p_{v_3} (\partial_\tau v_3)$$
$$\left. + p_{v_4} (\partial_\tau v_4) + p_{v_5} (\partial_\tau v_5) - \mathcal{H}_2^\mathrm{T} \Big] \right], \quad (40)$$

where the phase-space variables of the theory are $\Phi^i \equiv (X^\mu, A_0, A_1, b, \varphi, v_1, v_2, v_3, v_4, v_5)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_\mu, \pi^0, E, \Pi_b, \pi, p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}, p_{v_5})$. The functional measure $[\mathrm{d}\mu]$ of the generating functional $Z_2[J_i]$ under the GFCs ρ_i is obtained using (30), (32), (34) and (39):

$$\begin{aligned}
[\mathrm{d}\mu] = {} & [4M \delta''(\sigma - \sigma') \delta'(\sigma - \sigma') \delta^5 (\sigma - \sigma')] \\
& \times [\mathrm{d}X^\mu][\mathrm{d}A_0][\mathrm{d}A_1][[\mathrm{d}b][\mathrm{d}\varphi] \\
& \times [\mathrm{d}v_1][\mathrm{d}v_2][\mathrm{d}v_3][\mathrm{d}v_4][\mathrm{d}v_5] \\
& \times [\mathrm{d}\Pi_\mu][\mathrm{d}\pi^0][\mathrm{d}E][\mathrm{d}\Pi_b][\mathrm{d}\pi] \\
& \times [\mathrm{d}p_{v_1}][\mathrm{d}p_{v_2}][\mathrm{d}p_{v_3}][\mathrm{d}p_{v_4}][\mathrm{d}p_{v_5}] \\
& \times \delta[(\pi) \approx 0] \cdot \delta[(\Pi_b) \approx 0] \\
& \times \delta[(\pi^0) \approx 0] \cdot \delta(\Pi \cdot X') \approx 0] \\
& \times \delta[[\Pi^2 + (E^2 + T^2)(X')^2] \approx 0] \\
& \times \delta[(E) \approx 0] \cdot \delta[(X^2) \approx 0] \\
& \times \delta[(\Pi') \approx 0] \cdot \delta[(A_1) \approx 0] \cdot \delta[(b) \approx 0] \\
& \times \delta[(A_0) \approx 0] \cdot \delta[(\varphi) \approx 0]. \quad (41)
\end{aligned}$$

The Hamiltonian and path integral quantization of the theory described by the action S_2 under the GFCs ρ_i is now complete.

4 Summary and discussion

In this work we have studied the Hamiltonian and path integral quantization of the DBING action describing the D1-brane action with and without a scalar dilaton field φ under appropriate GFCs in the absence of BCs, using the instant form of dynamics on the hyperplanes of the WS defined by the hyperplanes WS-time $= \sigma^0 = \tau =$ constant. The problem of operator ordering occurring here while making a transition from EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Sect. 3, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators [6]. It is important to mention here that in our work we have not imposed any boundary conditions for the open and closed strings separately. There are two ways to take these BCs into account.

(a) One way is to impose them directly in the usual way for the open and closed strings separately in an appropriate manner [1,2];
(b) an alternative second way [7] is to treat these BCs as the Dirac primary constraints [7] and study the theory accordingly [7].

At present our related work is underway and will be reported later.

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